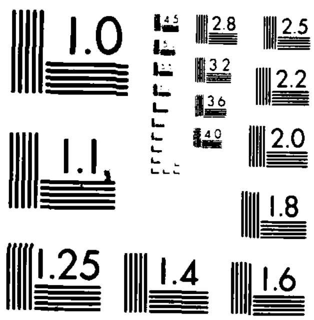
AD-A130 752 ROBUST ABLUE'S (ASYMPTOTICALLY BEST LINEAR UNBIASED ESTIMATORS) FOR LOCAT..(U) SOUTHERN METHODIST UNIV DALLAS TEX DEPT OF STATISTICS R L EUBANK ET AL. APR 83 1/1

UNCLASSIFIED TR-175 N00014-82-K-0207 F/G 12/1 NL

END
DATE FILMED
8 83
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

(12)

SOUTHERN METHODIST UNIVERSITY

AD A130752

DTIC
JUL 26 1983
A

DEPARTMENT OF STATISTICS

DALLAS, TEXAS 75275

This document has been approved for public release and sale; its distribution is unlimited.

DTIC FILE COPY

83 07 20 008

ROBUST ABLUE'S FOR LOCATION

AND SCALE PARAMETER ESTIMATION

by

R. L. Eubank and H. J. Lindsey

Technical Report No. 175
Department of Statistics ONR Contract

April, 1982

Research sponsored by the Office of Naval Research
Contract N00014-82-K-0207

Reproduction in whole or in part is permitted
for any purpose of the United States Government

The document has been approved for
public release and sale; its distribution is unlimited

DEPARTMENT OF STATISTICS
Southern Methodist University
Dallas, TX 75275

Robust ABLUE's for Location and Scale Parameter Estimation

by

R. L. Eubank and H. J. Lindsey
Southern Methodist University

Abstract: A robust procedure for location and/or scale parameter estimation is presented which utilizes the asymptotically best linear unbiased estimators (ABLUE's) based on $k(<N)$ of the N sample quantiles. Using regression design techniques a method is developed for selecting sample quantiles which furnishes the corresponding parameter estimates with good robustness properties relative to a given finite set of known probability laws. The problems of robust quantile selection for the estimation of a particular population quantile or in the presence of left and/or right hand censoring are also considered.

1. Introduction.

A location and scale parameter model assumes that a random sample, $X_1, \ldots, X_N$, has been obtained from a distribution of the form $F(\frac{x-\mu}{\sigma})$ where μ and σ are respectively location and scale parameters. An important problem associated with this model is the development of parameter estimates which behave robustly (e.g., insure high asymptotic relative efficiencies) over several or, perhaps, many choices for the form of F. In this paper we provide an approach for obtaining such estimators which utilizes the asymptotically best linear unbiased

estimators (ABLUE's) of μ and σ based on $k<N$ sample quantiles.

The ABLUE's are easily computed estimators whose properties have been extensively studied (c.f. Ogawa (1951), Sarhan and Greenberg (1962), Eubank (1981) and the references therein). Let $\tilde{Q}$ denote the sample quantile function defined by

$$\tilde{Q}(u) = X_{(j)}, \ \frac{j-1}{N} < u \le \frac{j}{N}, \ j = 1,\dots,N , \tag{1.1}$$

where $X_{(j)}$ is the jth sample order statistic; then given a <u>spacing</u>, $U = \{u_1,\dots,u_k\}$ (a set of k real numbers satisfying $0<u_1<u_2<\dots<u_k<1$), the ABLUE's are linear combinations of the corresponding sample quantiles $\tilde{Q}(u_1),\dots,\tilde{Q}(u_k)$. Explicit formulae for the coefficients for the $\tilde{Q}(u_i)$ may be found in Ogawa (1951) or Eubank (1979). As these coefficients are dependent on the form chosen for F, we adopt the notation $\hat{\mu}(U,F)$, $\hat{\sigma}(U,F)$ for the ABLUE's of μ and σ in order to indicate dependence on both the distribution and the spacing.

To measure the performance of an ABLUE one usually examines its asymptotic efficiency relative to the Cramér-Rao lower variance bound. Therefore, assume that F admits a density $f = F'$ and define the quantile function corresponding to F by

$$Q(u) = \inf\{x: F(x) \ge u\} .$$

The density-quantile function is then given by $fQ(u) = f(Q(u))$, $0 \le u \le 1$ (c.f. Parzen (1979) for a discussion of quantile and density-quantile functions and their properties). Using this notation the expressions derived by Ogawa (1951) for the asymptotic relative efficiencies (ARE's) for the ABLUE's may be written as follows:

i) The ARE for simultaneous estimation of μ and σ is

$$\mathrm{ARE}(\hat{\mu}(U,F),\ \hat{\sigma}(U,F)) = \frac{K_{11}(U,F)K_{22}(U,F)-K_{12}(U,F)^2}{|I(F)|} \tag{1.2}$$

where for $u_0=0, u_{k+1}=1$ and $fQ(0)=fQ(1)=fQ(0)Q(0)=fQ(1)Q(1)=0$ we have

$$K_{11}(U,F) = \sum_{i=1}^{k+1} \frac{[fQ(u_i) - fQ(u_{i-1})]^2}{u_i - u_{i-1}} \tag{1.3}$$

$$K_{22}(U,F) = \sum_{i=1}^{k+1} \frac{[fQ(u_i)Q(u_i) - fQ(u_{i-1})Q(u_{i-1})]^2}{u_i - u_{i-1}} \tag{1.4}$$

$$K_{12}(U,F) = \sum_{i=1}^{k+1} \frac{[fQ(u_i)-fQ(u_{i-1})][fQ(u_i)Q(u_i)-fQ(u_{i-1})Q(u_{i-1})]}{u_i - u_{i-1}} \tag{1.5}$$

with

$$I(F) = \begin{bmatrix} I_{\mu\mu}(F) & I_{\mu\sigma}(F) \\ I_{\mu\sigma}(F) & I_{\sigma\sigma}(F) \end{bmatrix} \tag{1.6}$$

denoting the usual Fisher information matrix for location and scale parameter estimation for the distribution F.

ii) The ARE for the estimation of μ when σ is known is

$$ARE(\hat{\mu}(U,F)) = K_{11}(U,F)/I_{\mu\mu}(F) \quad . \tag{1.7}$$

iii) The ARE for the estimation of σ when μ is known is

$$ARE(\hat{\sigma}(U,F)) = K_{22}(U,F)/I_{\sigma\sigma}(F) \quad . \tag{1.8}$$

Since these ARE's are dependent upon the spacing for the quantiles, U, it is common practice to further optimize by selecting U to maximize one of (1.2), (1.7) or (1.8) or, equivalently, their numerators $K_{11}(U,F)K_{22}(U,F)-K_{12}(U,F)^2$, $K_{11}(U,F)$ and $K_{22}(U,F)$. This is the so called <u>optimal spacing problem</u> and has been considered by numerous authors (for references see Hassanein (1977) and Eubank (1981)). We wish to consider, instead, the problem of selecting U's that are robust in the sense that they provide high ARE's over several possible choices for F.

The robust spacing selection technique presented in this paper is motivated by an approach to location parameter estimation developed by Chan and Rhodin (1980). They consider the problem of robustly estimating μ when F is assumed to belong to a finite set of probability laws, $\mathcal{L}$. For $L \in \mathcal{L}$ let U(L) denote the corresponding optimal (maximum ARE) k-element spacing; then, Chan and Rhodin suggest choosing a spacing, U(L*), which satisfies

$$\min_{G\in\mathcal{L}} \mathrm{ARE}(\hat{\mu}(U(L^*), G)) = \max_{L\in\mathcal{L}} \min_{G\in\mathcal{L}} \mathrm{ARE}(\mu(U(L),G)). \tag{1.9}$$

Thus, one selects the element of $\{U(L);L\in\mathcal{L}\}$ that is robust in the sense of providing the largest guaranteed ARE (GARE) of at least $\min_{G\in\mathcal{L}} \mathrm{ARE}(\hat{\mu}(U(L^*),G))$ regardless of which law in $\mathcal{L}$ generated the sample. In addition, this provides a candidate for F, namely L*, and, hence, μ may be estimated by $\mu(U(L^*),L^*)$.

In this paper regression design techniques are utilized to develop an asymptotic (as $k\to\infty$) version of the Chan and Rhodin procedure which, in contrast to their approach, has the advantages that it i) allows for the estimation of either or both of μ and σ and ii) does not require the tedious computation and tabulation of optimal spacings. Due to this latter quality, the procedure we propose is amenable to use with either large or small values of k. This fact is of particular importance since it will usually be necessary to use larger values of k to insure satisfactory GARE's than would be required merely for ARE consideration. We present our method for spacing selection in Section 2 accompanied with a discussion of its extension to the problems of quantile estimation and parameter estimation from singly or doubly censored samples. A numerical example is provided in Section 3.

2. Robust spacing selection

Parzen (1979) has shown that, for large N, location and scale parameter estimation can be considered as a regression analysis problem via the model

$$fQ(u)\tilde{Q}(u) = \mu fQ(u) + \sigma fQ(u)Q(u) + \sigma_B B(u),\ u \in [0,1]\ , \qquad (2.1)$$

where $\sigma_B = \sigma/\sqrt{N}$ and $B(\cdot)$ is a zero mean normal process with covariance kernel $\mathrm{Cov}(B(u_1), B(u_2)) = \min(u_1,u_2) - u_1u_2$. Eubank (1981) noted that the generalized least squares estimators of μ and σ obtained by sampling from this model at a design $U = \{u_1,\ldots,u_k\}$ whose elements satisfy $0<u_1<u_2<\ldots<u_k<1$ were, in fact, the ABLUE's. As a result, it was shown that problems of spacing selection could be phrased as regression design problems for model (2.1). The relationship between these two problems will be exploited, subsequently, to select spacings which have good robustness properties.

Our attention will be restricted to designs (spacings) that are obtained from continuous densities on [0,1]. Let h be such a density with associated distribution and quantile functions H and H^{-1}. Then, h <u>generates</u> a sequence of designs, $\{U_k\}$, with

$$U = \left\{H^{-1}\left(\frac{1}{k+1}\right),\ldots,H^{-1}\left(\frac{k}{k+1}\right)\right\}\ ,$$

i.e., $\{U_k\}$ is the design sequence whose kth element is composed of the (k+1)-tiles of H. For designs which are obtained in this manner it is possible to characterize the asymptotic (as $k\to\infty$) behaviour of the ARE's of the corresponding ABLUE's for the various estimation situations. It follows directly from the work of Sacks and Ylvisaker (1966, 1968) and Eubank (1981) that, under appropriate conditions on h:

i) When both μ and σ are unknown,

$$|I(F)| - |K(U_k,F)| = \frac{1}{12k^2} C_1(h,F) + o(k^{-2}) \tag{2.3}$$

where $K(U_k,F)$ is the matrix having ijth element $K_{ij}(U_k,F)$, $i,j=1,2$, and

$$C_1(h,F) = \int_0^1 \frac{\psi(u)^t I^{-1}(F)\psi(u)}{h(u)^2} du \tag{2.4}$$

with $\psi(u)$ defined by

$$\psi(u) = ([fQ(u)]'', [fQ(u)Q(u)]'')^t \quad . \tag{2.5}$$

ii) When only μ is unknown,

$$I_{\mu\mu}(F) - K_{11}(U_k,F) = \frac{1}{12k^2} C_2(h,F) + o(k^{-2}) \tag{2.6}$$

where

$$C_2(h,F) = \int_0^1 \frac{\{[fQ(u)]''\}^2}{h(u)^2} du \quad . \tag{2.7}$$

iii) When only σ is unknown

$$I_{\sigma\sigma}(F) - K_{22}(U_k,F) = \frac{1}{12k^2} C_3(h,F) + o(k^{-2}) \tag{2.8}$$

where

$$C_3(h,F) = \int_0^1 \frac{\{[fQ(u)Q(u)]''\}^2}{h(u)^2} du \quad . \tag{2.9}$$

These characterizations can be shown to hold, for instance, when the elements of ψ are continuous and $\int_0^1 \{h(u)\}^{-2} du < \infty$. Alternative conditions can be found in Sacks and Ylvisaker (1968, Theorem 3.1) whereas *somewhat* weaker restrictions may be deduced from the work of Pence and Smith (1981).

Equations (2.3), (2.6) and (2.8) have the consequence that, for

sufficiently large k, what distinguishes between the efficiency of spacings selected from various densities is their respective values for the asymptotic constants C_1, C_2 and C_3. In particular, one can construct an asymptotic solution to the optimal spacing problem by minimizing $C_i(h,F)$ over h for a given F. The solution was found by Eubank (1981) to be

$$h_F(u) = \begin{cases} \{\psi(u)^t I^{-1}(F)\psi(u)\}^{1/3}/\lambda_1 & \text{, when both } \mu \text{ and } \sigma \text{ are unknown,} \\ \{[fQ(u)]''\}^{2/3}/\lambda_2 & \text{, when only } \mu \text{ is unknown ,} \\ \{[fQ(u)Q(u)]''\}^{2/3}/\lambda_3 & \text{, when only } \sigma \text{ is unknown,} \end{cases} \tag{2.10}$$

where λ_1, λ_2 and λ_3 are appropriate normalizing constants.

The elements of the spacing sequence generated by h_F exhibit the same asymptotic behaviour as a sequence of optimal spacings in the sense that the ARE's for both sequences converge to the same limit (either $|I(F)|$, $I_{\mu\mu}(F)$ or $I_{\sigma\sigma}(F)$) at the same rate (namely $O(k^{-2})$). In fact the spacings obtained using (2.10) have been found to provide an excellent approximate solution to the optimal spacing problem even for k as small as 5 or 7. The reader is referred to Eubank (1979, 1981) for a discussion of the spacings generated by h_F and their properties.

The objective of this paper is to utilize the characterizations (2.3), (2.6) and (2.8) to provide a robust (as opposed to optimal) spacing selection scheme. To do so, we again utilize the constants $C_i(h,F)$. However, instead of optimizing over h for a fixed F, we will optimize with respect to both h and F as follows. Let $\mathcal{L}$ denote a finite set of probability laws and assume that for all combinations of $L,G\varepsilon\mathcal{L}$ the constant $C_i(h_L,G)$ for the estimation problem of interest is well defined. An approximate (for large k) solution to the problem of robust spacing selection is then given by: Choose $L^*\varepsilon\mathcal{L}$ so that

$$\max_{G\varepsilon\mathcal{L}} C_i(h_{L^*},G) = \min_{L\varepsilon\mathcal{L}} \max_{G\varepsilon\mathcal{L}} C_i(h_L,G) \quad . \tag{2.11}$$

Depending upon which parameters are to be estimated, one then uses either $(\hat{\mu}(U_k^*, L^*), \hat{\sigma}(U_k^*, L^*))^t$, $\hat{\mu}(U_k^*, L^*)$, or $\hat{\sigma}(U_k^*, L^*)$ for estimation purposes where U_k^* is the nth element of the design sequence generated by h_{L^*}. We observe that in the case of location parameter estimation ($i = 2$) equation (2.11) can be viewed as providing an asymptotic version of (1.9).

An illustration of the use of (2.11) is provided in the next section. First, however, we discuss some of the computational details and merits of this approach as well as certain of its extensions to other robust estimation problems.

Computation of the constants (2.4), (2.7) and (2.9) will usually require numeric integration. For this purpose the use of a Gaussian quadrature rule is recommended. One then constructs a table consisting of the values of $C_i(h_L, G)$ for all combinations of L and G in $\mathcal{L}$ from which a law satisfying (2.11) can be readily found. This table has the same role in our procedure as that of Tables 3,4,5 and 6 (i.e., tabulations of the ARE's for optimal spacings for all L,G combinations corresponding to $k = 2,3,4$ and 5 respectively) in the Chan and Rhodin method. However, due to the asymptotic nature of our solution a decision reached using (2.11) obtains for all k. The computational savings derived from this technique are, therefore, twofold in that i) a single table suffices for all values of k and ii) the computation of the constants C_1, C_2 and C_3 requires only numeric integration as compared to the solution of nonlinear equation systems required by procedures based on optimal spacings (c.f. Sarhan and Greenberg (1962) for examples of such equations). Consequently, criterion (2.11) is amenable to use with larger values of k. The importance of this fact should not be over-

looked as, in order to insure high GARE's, it will be necessary to use ABLUE's based on more quantiles than would be needed merely to obtain high ARE's.

Once an L* satisfying (2.11) has been found the next step is to compute the corresponding k-point design, U_k^*. Although the design density h_{L*} will frequently not have a closed form this can still be easily accomplished through numeric tabulation of H_{L*} followed by interpolation to find the design points $H_{L*}^{-1}(\frac{i}{k+1})$, i=1,...,k. The corresponding estimator coefficients are then computed in the usual manner using the formulae found in either Ogawa (1951) or Eubank (1979). It should be noted that whereas the spacings selected using (2.11) are robust this does not imply that the use of coefficients corresponding to L* is an optimal strategy for robustness considerations (similar comments also apply to the Chan and Rhodin method). In practice one may wish to use the spacings for L* in conjunction with coefficients for other members of $\mathcal{L}$ or, perhaps, employ some type of averaging procedure. The development of a robust coefficient selection scheme to accompany (2.11) (as well as (1.9)) is currently an open research problem.

The robust spacing selection techniques presented here can be adapted to provide solutions to other robust estimation problems. For instance, it is sometimes difficult to interpret the parameter comparisons that are implied by (2.11). Such difficulties can be averted by comparing the various location and scale parameter models for the laws in $\mathcal{L}$ on the basis of one or more specific quantiles. Let τ denote a specified percentile point and for $L \varepsilon \mathcal{L}$ let Q_L denote the corresponding quantile function. The τth quantile for the model $L(\frac{x-\mu}{\sigma})$ is

then $\mu + \sigma\, Q_L(\tau)$ for which an estimator is $\hat{\mu}(U,L) + \hat{\sigma}(U,L)Q_L(\tau)$. The asymptotic variance of this estimator is found to be proportional to the trace(tr) of $K^{-1}(U,L)v_\tau(L)v_\tau(L)^t$ where $v_\tau(L) = (1,\ Q_L(\tau))^t$. In a manner similar to previous developments we have , under certain additional conditions on h (c.f. Theorem 4.5 of Sacks and Ylvisaker (1968)), that

$$\mathrm{tr}([I^{-1}(L)-K^{-1}(U_k,L)]v_\tau(L)v_\tau(L)^t) = \frac{1}{12k^2}\, C_4(h,L) + o\,(n^{-2}) \qquad (2.12)$$

with

$$C_4(h,L) = \int_0^1 \frac{\psi(u)^t I^{-1}(L)v_\tau(L)v_\tau(L)^t I^{-1}(L)\psi(u)}{h(u)^2}\, du \quad . \qquad (2.13)$$

An optimal density for this problem is proportional to

$$\{\psi(u)^t I^{-1}(L)v_\tau(L)v_\tau(L)^t I^{-1}(L)\psi(u)\}^{1/3}$$

and, hence, (2.13) can now be used in (2.11) to determine robust spacings for the purpose of quantile estimation.

It is also possible to modify (2.11) to provide robust spacings for problems of estimation from left and/or right censored samples such as those considered by Cheng (1980). This can be accomplished through use of the optimal densities for spacing selection from censored samples given in Eubank (1981).

3. Numerical example.

As an illustration of the use of the spacing selection technique provided in Section 2 consider the data consisting of lifetimes for 417 40-watt internally frosted incandescent lamps presented in Davis (1952). This data has also been analyzed by Chan and Rhodin (1980) under the assumption that the parent distribution was either a member

of (or well approximated by) the Tukey lambda family of distributions or was a normal, double exponential or Cauchy distribution. Through goodness of fit considerations they chose L to consist of the normal distribution and those members of the Tukey lambda family having shape parameters -.1, 0, .1 and .14. To facilitate comparison with their results L will be chosen similarly here. Letting $L(\lambda)$ denote the Tukey lanbda distribution with shape parameter λ, we take $L=\{L(-.1),L(0), L(.1), L(.14)\}$ where, for simplicity, the normal has been omitted since it is approximated by L(.14).

The quantile function for $L(\lambda)$ is

$$Q_\lambda(u) = \lambda^{-1}[u^\lambda + (1-u)^\lambda]$$

with associated density-quantile function

$$(fQ)_\lambda(u) = [u^{\lambda-1} + (1-u)^{\lambda-1}]^{-1}.$$

Thus, the optimal density, $h_{L(\lambda)} = h_\lambda$, for locatic.. parameter estimation is proportional to

$$\{(fQ_\lambda)''(u)\}^{2/3}=(u^{\lambda-1}+(1-u)^{\lambda-1})^{-2}\{(\lambda-1)\lambda[u^{\lambda-2}-(1-u)^{\lambda-2}]^2-(\lambda-1)(\lambda-2)[u(1-u)]^{\lambda-3}\}^{2/3}.$$

As $h_\lambda(u) = h_\lambda(1-u)$, the spacings generated by h_λ will be symmetric, i.e., $u_i = 1-u_{k+1-i}$. This has the consequence that, $\hat{\mu}(U_k, L(\lambda))$ is scale invariant (c.f. also Chan and Rhodin (1980) pg. 228 for further comments on this property).

The constants $C_2(h_\lambda, L(\eta)) = C_2(\lambda,\eta)$ have the form

$$C_2(\lambda,\eta) = \int_0^1[(fQ_\eta)''(u)/h_\lambda(u)]^2du.$$

The values of $C_2(\lambda,\eta)$ have been tabulated for all $\lambda,\eta \in \{-.1,0,.1,.14\}$ and are provided in Table 1. It follows, upon examination of Table 1,

TABLE 1

The values of $C_2(\lambda,\eta)$ for $\lambda,\eta \in \{-.1,0,.1,.14\}$

λ \ η	-.10	0	.10	.14
-.10	3.378	45.908	243.438	369.574
0	26.0108	4.000	297.194	916.012
.10	5.596	6.311	10.895	16.225
.14	6.973	7.759	11.445	15.312

that the minimum value of $\max_{\eta} C_2(\lambda,\eta)$ occurs at $\lambda^* = .14$ and, hence, $h_{.14}$ generates spacings that are robust relative to L. It is of interest to note that this choice of λ agrees with the one made by Chan and Rhodin for k=2 and is their second choice (i.e. has the second largest minimum ARE) when k=3. It is unfortunate that tables are not available for larger values of k, such as k= 7,9 or perhaps even 20, where one might expect the results from these two techniques to consistently coincide.

Numeric tabulation of the density $h_{.14}$ reveals that the robust 5 element spacing is $U_5 = \{.0105, .1628, .5, .8372, .9895\}$ from which the corresponding estimator is computed to be (using Ogawa's formula)

$$\begin{aligned}\hat{\mu}(U_5,L(.14)) &= .0396[\tilde{Q}(.0105)+\tilde{Q}(.9895)] + .2585[\tilde{Q}(.1628)+\tilde{Q}(.8372)] \\ &\quad + .4037\tilde{Q}(.5) \\ &= .0396[609+1550] + .2585[883 + 1225] + .4037[1037] \\ &= 1049.0513.\end{aligned}$$

This is to be compared with the value 1044.3 obtained by Chan and Rhodin (1980) also through the use of a 5-quantile ABLUE.

As larger values of k will usually be required to insure sufficiently high GARE's it is important to note that once $h_{.14}$ has been tabulated spacings of any desired size can be easily computed. For instance, when k = 7 the robust spacing is $U_7 = \{.0034, .0574, .2353, .5, .7647, .9426, .9966\}$ with corresponding coefficients $\{.0148, .0995, .2374, .2967, .2374, .0995, .0148\}$. The resulting estimator is then found to be $\hat{\mu}(U_7,L(.14)) = 1045.97$.

It is also of interest to ascertain how U_5 and U_7, perform in terms of GARE. Consequently, the ARE's for these spacings have been computed over the various laws in L and are provided in Table 2. The minimum or guaranteed ARE when $k = 5$ is found to be .908 whereas when $k = 7$ the GARE is .9466. Although the set L considered by Chan and Rhodin includes the normal while ours does not, it is still noteworthy that the GARE for U_5 is quite similar to the GARE of .9015 for the robust 5-quantile ABLUE based on optimal spacings utilized by Chan and Rhodin.

TABLE 2

ARE's for U_5 and U_7 for the various laws in $\mathcal{L}$.

η	$ARE(\hat{\mu}(U_5,L(\eta)))$	$ARE(\hat{\mu}(U_7,L(\eta)))$
-.1	.9080	.9466
0	.9163	.9513
.1	.9156	.9506
.14	.9111	.9474

REFERENCES

Chan, L.K. and Rhodin, L.S. (1980). Robust estimation of location using optimally chosen sample quantiles. Technometrics 22, 225-237.

Cheng, S.W. (1980). On the ABLUE of the normal mean from a censored sample. J. Statist. Planning and Inference 4, 259-265.

Davis, D.J. (1952). An analysis of some failure data. J. Amer. Statist. Assoc. 47, 113-150.

Eubank, R. L. (1979). A density-quantile function approach to the selection of order statistics for location and scale parameter estimation. Tech. Rep. No. A10, Institute of Statistics, Texas A & M Univ.

Eubank, R. L. (1981). A density-quantile function approach to optimal spacing selection. Ann. Statist. 9, 494-500.

Hassanein, K.M. (1977). Simultaneous estimation of the location and scale parameter of the gamma distribution by linear functions of order statistics. Skand. Aktuarietidskr. 56, 120-128.

Ogawa, J. (1951). Contributions to the theory of systematic statistics. Part I. Osaka Math. J. 3, 175-213.

Parzen, E. (1979). Nonparametric statistical data modeling. J. Amer. Statist. Assoc. 74, 105-131.

Pence, D.D. and Smith, P.W. (1981). Asymptotic properties of best $L_p[0,1]$ approximation by splines. SIAM J. Math. Anal. 13, 409-420.

Sacks, J. and Ylvisaker, D. (1966). Designs for regression problems with correlated errors. Ann. Math. Statist. 37, 66-89.

Sacks, J. and Ylvisaker, D. (1968). Designs for regression problems with correlated errors; many parameters. Ann. Math. Statist. 39, 49-69.

Sarhan, A.E. and Greenberg, B. G. (eds.) (1962). Contributions to Order Statistics. Wiley, New York.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 175	2. GOVT ACCESSION NO. AD-A130 152	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ROBUST ABLUE'S FOR LOCATION AND SCALE PARAMETER ESTIMATION		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER 175
7. AUTHOR(s) R. L. Eubank and H. J. Lindsey		8. CONTRACT OR GRANT NUMBER(s) N00014-82-K-0207
9. PERFORMING ORGANIZATION NAME AND ADDRESS Southern Methodist University Dallas, TX 75275 and University of Delaware, Newark, Delaware 19711		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR 042-479
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Arlington, VA 22217		12. REPORT DATE April, 1983
		13. NUMBER OF PAGES 16
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report)
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

This document has been approved for public release and sale; its distribution is unlimited. Reproduction in whole or in part is permitted for any purposes of the United States Government.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Asymptotically Best Linear Unbiased Estimator; Asymptotic Relative Efficiency; Robust Spacings; Robust Designs; Location and Scale Parameters.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A robust procedure for location and/or scale parameter estimation is presented which utilizes the asymptotically best linear unbiased estimators (ABLUE's) based on k(N) of the N sample quantiles. Using regression design techniques a method is developed for selecting sample quantiles which furnishes the corresponding parameter estimates with good robustness properties relative to a given finite set of known probability laws. The problems of robust quantile selection for the estimation of a particular population quantile or in the presence of left and/or right hand censoring are also considered.

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

END

DATE

FILMED

– 83

DTIC